Digital Circuits

For

EC / EE / IN

By

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Syllabus for Digital Circuits

Number Systems, Combinatorial Circuits, Boolean Algebra, Minimization of Functions using Boolean Identities and Karnaugh Map, Logic Gates and their Static CMOS Implementations, Arithmetic Circuits, Code Converters, Multiplexers, Decoders and PLAs, Sequential Circuits Latches and Flip-Flops, Counters, Shift-Registers and Finite State Machines, Data Converters, Sample and Hold Circuits, ADCs and DACs, Semiconductor Memories, ROM, SRAM, DRAM, 8-bit Microprocessor (8085), Architecture, Programming, Memory and I/O Interfacing.

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"The best way to predict your future is to create it."
– **Abraham Lincoln**

CHAPTER 1 Number Systems & Code Conversions

Learning Objectives

After reading this chapter, you will know:

1. Base or Radix System
2. System Conversions, Coding Techniques
3. Binary Arithmetic
4. BCD Addition
5. Complements

Introduction

The concept of counting is as old as the evolution of man on this earth. The number systems are used to quantify the magnitude of something. One way of quantifying the magnitude of something is by proportional values. This is called analog representation. The other way of representation of any quantity is numerical (Digital). There are many number systems present. The most frequently used number systems in the applications of Digital Computers are Binary Number System, Octal Number System, Decimal Number System and Hexadecimal Number System.

Base or Radix (r) of a Number System

The Base or Radix of a number system is defined as the number of different symbols (Digits or Characters) used in that number system.

The Radix of Binary number system = 2, i.e., it uses two different symbols 0 and 1 to write the number sequence.

The Radix of Octal number system = 8, i.e., it uses eight different symbols 0, 1, 2, 3, 4, 5, 6 and 7 to write the number sequence.

The Radix of Decimal number system = 10, i.e., it uses ten different symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to write the number sequence.

The Radix of Hexadecimal number system = 16, i.e., it uses sixteen different symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,A, B, C, D, E and F to write the number sequence.

The Radix of Ternary number system = 3, i.e., it uses three different symbols 0, 1 and 2 to write the number sequence.

To distinguish one number system from the other, the radix of the number system is used as suffix to that number.

E.g.: $(10)_2$ Binary Number; $(10)_8$ Octal Number;
$(10)_{10}$ Decimal Number; $(10)_{16}$ Hexadecimal Number;

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Characteristics of any Number System are

1. Base or radix is equal to the number of unique single digits in the system.
2. The largest value of digit is one (1) less than the radix, and the maximum value of digit in any number system is given by $(\Omega - 1)$, where Ω is radix.
3. Each digit is multiplied by the base raised to the appropriate power depending upon the digit position.

   E.g.: Maximum value of digit in decimal number system = (10 – 1) = 9.

Positional Number Systems

In a positional number systems there is a finite set of symbols called digits. Each digits having some positional weight. Below table shows some positional number system and their possible symbols

Number System	Base	Possible Symbols
Binary	2	0, 1
Ternary	3	0, 1, 2
Quaternary	4	0, 1, 2, 3
Quinary	5	0, 1, 2, 3, 4
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Duodecimal	12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

- Binary, Octal, Decimal and Hexadecimal number systems are called positional number systems.
- Any positional number system can be expressed as sum of products of place value and the digit value.

  E.g.: $(\mathbf{756})_{\mathbf{10}} = 7 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

  $(\mathbf{156.24})_8 = 1\times 8^2 + 5 \times 8^1 + 6 \times 8^0 + 2 \times 8^{-1} + 4 \times 8^{-2}$

Decimal Point

- The place values or weights of different digits in a mixed decimal number are as follows:

  $10^5 \quad 10^4 \quad 10^3 \quad 10^2 \quad 10^1 \quad 10^0 \;.\; 10^{-1} \quad 10^{-2} \quad 10^{-3} \quad 10^{-4}$

Binary Point

- The place values or weights of different digits in a mixed binary number are as follows:

  $2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \;.\; 2^{-1} \quad 2^{-2} \quad 2^{-3} \quad 2^{-4}$

Octal Point

- The place values or weights of different digits in a mixed octal number are as follows:

  $8^4 \quad 8^3 \quad 8^2 \quad 8^1 \quad 8^0 \;.\; 8^{-1} \quad 8^{-2} \quad 8^{-3} \quad 8^{-4}$

Hexadecimal Point

- The place values or weights of different digits in a mixed Hexadecimal number are as follows:

  $16^4 \quad 16^3 \quad 16^2 \quad 16^1 \quad 16^0 \;.\; 16^{-1} \quad 16^{-2} \quad 16^{-3} \quad 16^{-4}$

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System Conversion

Decimal to Binary Conversion

(a) **Integer Number:** Divide the given decimal integer number repeatedly by 2 and collect the remainders. This must continue until the integer quotient becomes zero.

E.g.: $(37)_{10}$

Operation	Quotient	Remainder
37/2	18	+1
18/2	9	+0
9/2	4	+1
4/2	2	+0
2/2	1	+0
1/2	0	+1

1 0 0 1 0 1

$\therefore (37)_{10} = (100101)_2$

Note: The conversion from decimal integer to any base-r system is similar to the above example except that division is done by r instead of 2.

(b) **Fractional Number**: The conversion of a Decimal fraction to a Binary is as follows:

E.g.: $(0.68755)_{10} = X_2$

First, 0.6875 is multiplied by 2 to give an integer and a fraction. The new fraction is multiplied by 2 to give a new integer and a new fraction. This process is continued until the fraction becomes 0 or until the numbers of digits have sufficient accuracy.

E.g.:	Integer value
$0.6875 \times 2 = 1.3750$	1
$0.3750 \times 2 = 0.7500$	0
$0.7500 \times 2 = 1.5000$	1
$0.5000 \times 2 = 1.0000$	1

$\therefore (0.6875)_{10} = (0.1011)_2$

Note: To convert a decimal fraction to a number expressed in base r, a similar procedure is used. Multiplication is done by r instead of 2 and the coefficients found from the integers range in value from 0 to (r−1).

- The conversion of decimal number with both integer and fraction parts are done separately and then combining the answers together.

E.g.: $(41.6875)_{10} = X_2$

$(41)_{10} = (101001)_2$

$(0.6875)_{10} = (0.1011)_2$

Since, $(41.6875)_{10} = (101001.1011)_2$.

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E.g.: Convert the Decimal Number to its Octal equivalent: $(153)_{10} = X_8$

Integer Quotient	Remainder
153/8	+1
19/8	+3
2/8	+2

$\therefore (153)_{10} = (231)_8$

Convert the Decimal to its octal equivalent

E.g.: $(0.513)_{10} \quad = X_8$

$0.513 \times 8 \quad = 4.104$
$0.104 \times 8 \quad = 0.832$
$0.832 \times 8 \quad = 6.656$
$0.656 \times 8 \quad = 5.248$
$0.248 \times 8 \quad = 1.984$
$0.984 \times 8 \quad = 7.872$
$\vdots$
$(153)_{10} \quad = (0.406517 \ldots)_8$

E.g.: Convert $(253)_{10}$ to Hexadecimal

$253/16 = 15 + (13 = D)$
$15/16 \ = 0 + (15 = F)$
$\therefore (253)_{10} = (FD)_{16}$

E.g.: Convert the Binary number 1011012 to Decimal.

$101101 = 2^5 \times 1 + 2^4 \times 0 + 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 0 + 2^0 \times 1$
$= 32 + 8 + 4 + 1 = 45$
$(101101)^2 = (45)_{10}$

E.g.: Convert the Octal number $(257)_8$ to Decimal.

$(257)_8 = 2 \times 8^2 + 5 \times 8^1 + 7 \times 8^0$
$= 128 + 40 + 7 = (175)_{10}$

E.g.: Convert the Hexadecimal number 1AF.23 to Decimal.

$(1AF.23)_{16} = 1 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 + 2 \times 16^{-1} + 3 \times 16^{-2} = 431.1367$

Important Points

1. A Binary will all 'n' digits of '1' has the value $2^n - 1$
2. A Binary with unity followed by 'n' zero has the value 2^n. It is an $n + 1$ digit number

E.g.:

(a) Convert Binary 11111111 to its Decimal value
Solution: All eight bits are unity. Hence value is $2^8 - 1 = 255$

(b) Express 2^{10} as Binary
Solution: 2^{10} is written as unity followed by 2^{10} zero, 10000000000

Same rule apply for other number code

E.g.: Express 8^4 in Octal system

Solution: $8^1 = (10)_8 \qquad 8^3 = (1000)_8$
$8^2 = (100)_8 \qquad 8^4 = (10000)_8$

Solution: $8^4 = (10000)_8$

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Binary to Decimal Conversion (Short Cut Method)

Binary to Decimal = Binary → Octal → Decimal

E.g.: Convert 101110 into Decimal

Solution: $(\underbrace{101}_{5}\,\underbrace{110}_{6})_2 = (56)_8 = 5 \times 8 + 6 = (46)_{10}$

Note: For Converting Binary to Octal make group of 3 bits starting from left most bit.

Binary to Decimal Conversion (Equation Method)

$S_i = 2S_{i-1} + a_{i-1}$

Where $S_n = a_n$ and S_0 → The last sum term

E.g.: $(1101)_2$ to Decimal

1	1	0	1
↓	+	+	+
1 ×2	2 ×2	6 ×2	12
1	3	6	13

So $(1101)_2 = (13)_{10}$

Note: We can use calculator (Scientific) but there is a limit in number of digit as input in calculator. We can use transitional way of multiplying each digit with 2^{n-1} (Where n is the Position of Digit in Binary Number) and adding in the last but for large binary digit its again a tedious task

E.g.: $(11101011110)_2$ to Decimal

1	1	1	0	1	0	1	1	1	1	0
↓	+ 2	+ 6	+ 14	+ 28	+ 58	+ 116	+ 234	+ 470	+ 942	+ 1886
1 (× 2)	3 (× 2)	7 (× 2)	14	29	58	117	235	471	943	1886

So $(11101011110)_2 = (1886)_{10}$

Octal to Decimal Conversion (Equation Method)

Above equation can be used for Octal to Decimal conversion with small modification.

$S_i = 8S_{i-1} + a_i$

E.g.: Convert $(3767)_8$ to Decimal

3	7	6	7
↓	+	+	+
×8	24 ×8	248 ×8	2032
3	31	254	2039

$(3767)_8 = (2039)_{10}$

Note: In general recursive equation to convert an integer in any base to base 10 (Decimal) is

$S_i = bS_{i-1} + a_i$

Where b → Base of the integer.

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Binary Fraction to Decimal

Since conversion of fractions from Decimal to other bases requires multiplication. It is not surprising that going from other bases to Decimal required a division process

Binary Fraction	Division Process	
1	2	1.0
1	2	1.5
0	2	0.75
(MSB) 1	2	1.375
		0.6875

Hence $(0.1011)_2 = (0.6875)_{10}$

Another method: The Decimal value without Decimal point of 1011 is 11. For 4 bits. The number of combination is $2^4 = 16$. Hence Decimal value is $\frac{11}{16} = 0.6875$

Coding Techiniques

Binary Code: First let us have a brief study about codes. If we want to place the number 14_{10} into a computer, we could write $(1110)_2$, or the individual digits of the decimal number could be coded separately as 0001 0100. The second method is called "Binary-Coded Decimal". BCD has been used in some commercial computers because of it's similarity to the Decimal number system.

One should distinguish between the terms "Conversion" and "Encoding". "Conversion" is the more drastic process in that the structure of the number itself is changed **E.g.,** a different base is used. Encoding keeps the basic structure of the number the same, but the individual digits are represented by different symbols.

E.g.: How many bits (Binary Digits) are needed to code the ten symbols 0-9?

A solution is desired for the equation $2^n = 10$, where n is the required number of bits. Using a calculator or log-table, we find that $n = \log_2 10 = 3.01$; but since n must be an integer, four bits are needed. However $2^4 = 16$; Thus there are $16-10 = 6$ Excess symbols. Further, by using 10 of 16 symbols, the number of different codes we can construct are, $16P_{10}\ \frac{16!}{6!} \approx 3 \times 10^{10}$ codes.

Some typical BCD codes are shown in Table below.

Typical BCD Codes				
Decimal	Common Binary (8, 4, 2, 1)	Excess – 3	Gray code	Code X
0	0000	0011	0000	0100
1	0001	0100	0001	0010
2	0010	0101	0011	0111
3	0011	0110	0010	1110
4	0100	0111	0110	0011
5	0101	1000	0111	1100
6	0110	1001	0101	0001
7	0111	1010	0100	1000
8	1000	1011	1100	1101
9	1001	1100	1101	1011

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